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MAGMA CHAMBER DEFLATION RECORDED BY THE GLOBAL POSITIONING SYSTEM: THE HEKLA 1991 ERUPTION

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Abstract. Between January 17 and March 11, 1991, 0.15 km^3 of lava erupted initially from several radial fissures and subsequently from a single fissure on the SE flank of Hekla volcano, Iceland. Hekla is surrounded by an array of control points measured in 1989 using GPS geodesy and re-measured after the eruption. These measurements indicate that the eruption was associated with a surface deflation volume of $0.1^{+0.08}_{-0.04}$ centered on Hekla (63.995°N $^{+4}_{-3}$ km, 19.69°W $^{+1.5}_{-2}$ km). The depth to the magma reservoir is 9^{+6}_{-7} km, poorly constrained due to the absence of GPS control points close to the volcano.

Introduction

The volcano Hekla is located near the intersection of the Eastern Volcanic Zone and the transform zone in South Iceland (Figure 1). The volcanic history since 1104, the first documented eruption, is characterized by one or two powerful eruptions per century. Yet the 1991 eruption is the fourth Hekla eruption this century. The vigorous 1947-48 eruption was the latest typical eruption of Hekla [*Thorarinsson*, 1967], preceded by a repose period of 102 years. Since then 3 unexpected eruptions have occurred, in 1970, 1980-81 and in 1991 [*Thorarinsson and Sigvaldason*, 1972; *Gronvold et al.*, 1983; *Gudmundsson et al.*, 1992]. The 1991 eruption from January 17 to March 11 produced 0.15 km^3 of basaltic andesite lava. Initially several fissures with a radial pattern were active, but already on the second day activity was mainly confined to a single fissure on the SE flank of the volcano where the main crater subsequently formed (Figure 2). The effusive activity was most vigorous during the first few days of the eruption.

In 1986, Global Positioning System [*Leick*, 1990] receivers were used to establish a crustal deformation geodetic network in Iceland. In August 1989 six TI4100 receivers were used to reoccupy and densify the network [*Hackman*, 1991]. Highest priority was given to measurements in the South Iceland Seismic Zone where a M7 earthquake sequence is expected soon, thus measurements within the volcanic zones had lower priority. The rugged terrain near Hekla also made it difficult to establish GPS control close to the volcano. The nearest GPS control point is at a distance of 13 km from the summit, making the network far from ideal to study the deformation mechanisms of Hekla. However, the far-field observations of crustal deformation presented here provide important constraints on the eruptive mechanism.

Data

Three weeks after the beginning of the Hekla eruption re-measurements of nearby GPS control points were initiated using 3 Trimble 4000 SST receivers. From February 4 to 18, 1991 9 control points near Hekla were re-measured. In the first week of August 1991 3 Ashtech MD-XII receivers were used to re-measure 2 additional control points to the northeast of Hekla that were inaccessible in February. At the same time 5 of the points occupied in February were occupied again. In 1991 each point was occupied for at least 3 five-hour observing sessions, and in 1989 each point was occupied for at least 2 five-hour sessions. The control point at Ísakot was occupied in every observing session and later held fixed in the data processing. For each data set a network solution was calculated using broadcast orbits with version 3.3 of the Bernese GPS software [*Rothacher et al.*, 1990]. Phase ambiguities were not resolved to integers (which is, however, important for highest accuracy). The scatter of independently estimated results from each session is used to estimate the uncertainty in the components of locations relative to the control point at Ísakot as 0.9 - 2.4 cm (Table 1). Adverse ionospheric conditions introduce high frequency noise in the Icelandic GPS data, causing larger uncertainty than obtained under optimal measurement conditions where subcentimeter accuracy is obtained [*Larson and Agnew*, 1991]. Displacements from 1989 to 1991 are estimated by subtracting the 1989 location of the control points from the 1991 location. The uncertainty in the difference is about 2.5 cm for horizontal components (Table 1), the value used when modeling the data. The vertical uncertainty is >6 cm, and provides only maximum deformation constraints of little value in the current study.

Comparison of the 1991 and 1989 data sets reveals 3.1-8.8 cm horizontal displacements relative to the base control point at Ísakot (Figure 3a). Knowledge of the displacement at Ísakot

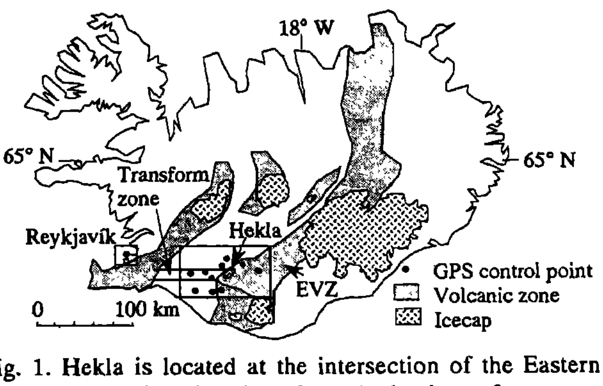

Fig. 1. Hekla is located at the intersection of the Eastern Volcanic Zone (EVZ) and the South Iceland transform zone. Boxes denote the location of Figure 3.

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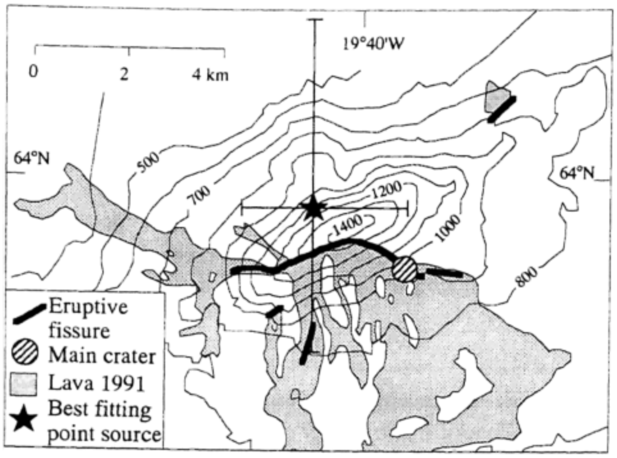

Fig. 2. Hekla volcano, eruptive fissures and the 1991 lava. The star with 1σ error bars represents the magma chamber location that best fits the GPS data. Elevation contours are at 100 m intervals. Map after *Gudmundsson et al.* [1992].

allows the absolute displacements at all control points to be deduced. An estimate of the displacement at Ísakot, $u_{Ísakot}$, is provided by our model as explained below ($u_{Ísakot}$ = 4.9 cm at 169°) or by assuming that the control point in Reykjavík (farthest away from Hekla) provides a stable reference and the absolute displacement there is zero ($u_{Ísakot}$ = 4.4 cm at 156°). The absolute displacements reveal radial displacements towards the volcano (Figure 3b). Data from stations common to the February and August 1991 surveys indicate insignificant

	Mean 1σ of daily estimates (cm)		
	1989	February 1991	August 1991
North	1.8	1.1	1.7
East	0.9	2.4	1.3
Length	1.1	1.4	1.2
Height	3.4	5.1	6.3
	Estimated difference uncertainty (cm)		
	February 1991 - 1989	August 1991 - 1989	August 1991 - February 1991
North	2.1	2.5	2.0
East	2.6	1.6	2.7
Length	1.8	1.6	1.8
Height	6.1	7.2	8.1

Table 1. Estimated mean 1σ uncertainty in the components of locations relative to the control point at Ísakot for each survey and difference uncertainty. For each component 1σ is the weighted RMS scatter about the weighted mean $<y>$ of the daily estimates y_i, given by

$$\sigma^2 = \frac{N}{N-1}\left[\sum_{i=1}^{N}\frac{(y_i - <y>)^2}{\sigma_i^2}\right]\left[\sum_{i=1}^{N}\frac{1}{\sigma_i^2}\right]^{-1}$$

where σ_i are the formal standard errors and N is the number of days each control point was occupied. The values in the table are the mean for all the control points. The difference uncertainty is given by $\sigma_{diff}^2 = \sigma_a^2 + \sigma_b^2$ where σ_a and σ_b are the uncertainty of the first and second survey respectively.

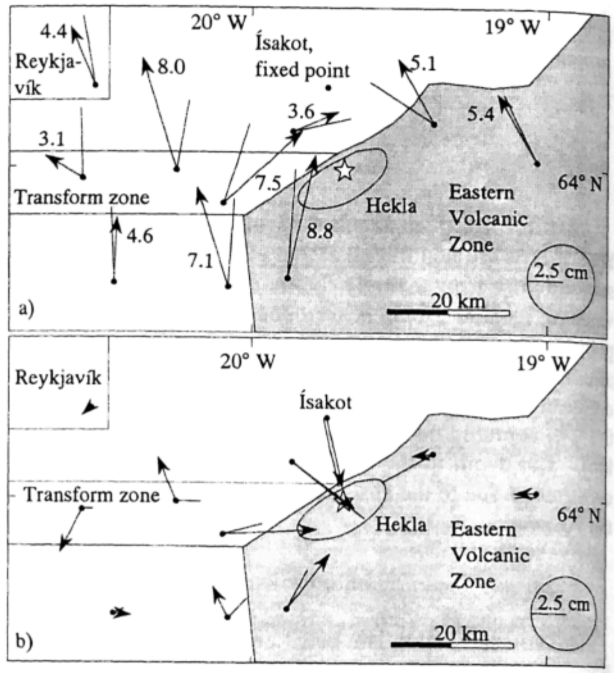

Fig. 3. a) Horizontal displacements of control points (dots) relative to the control point at Ísakot: observed (lines with arrows) and best model (lines). Numbers next to arrows are the measured relative displacements in cm. Displacement uncertainty is 2.5 cm (bottom right). The location of the control point in Reykjavík is shown in Figure 1. Hekla volcano is outlined and the inferred location of the magma chamber is indicated by a star. b) Absolute displacements assuming the control point at Ísakot moves 4.9 cm at 169° as the best fitting Mogi model predicts.

deformation in the time interval between the surveys. This allows us to use the February data and the data from the two control points first re-occupied in August as one data set when modeling the deformation.

Modeling

We fit a model to the relative horizontal displacements (in Figure 3a) using a least squares criteria minimizing the χ^2 merit function [*Bevington*, 1969, p. 205]

$$\chi^2 = \sum_{i=1}^{N}\left(\frac{\delta\mathbf{r}_i^{obs}(\mathbf{r}_i) - \delta\mathbf{r}_i^{pre}(\mathbf{r}_i;\mathbf{a})}{\sigma_i}\right)^2 \tag{1}$$

where $\mathbf{r}_i$ is the horizontal location (latitude and longitude) of control point i, $\delta\mathbf{r}_i^{obs}(\mathbf{r}_i)$ is the observed horizontal displacement vector at location $\mathbf{r}_i$, $\delta\mathbf{r}_i^{pre}(\mathbf{r}_i;\mathbf{a})$ is the predicted displacement from the model, $\mathbf{a}$ is the model parameter vector, σ_i is the uncertainty in $\delta\mathbf{r}_i^{obs}(\mathbf{r}_i)$ and N is the number of displacement vectors. We occupied eleven control points, and have used one as a reference, so N = 10. The best fitting model parameters, $\mathbf{a}^{est}$, are those that lead to a minimum value for χ^2. Assuming measurement errors to be normally distributed then χ^2 is distributed as a chi-square distribution with 2N-M degrees of freedom, 2N being the number of vector components we model and M being the number of adjustable model parameters. We first test the hypothesis that there has been no deformation, $\delta r_i^{pre}(\mathbf{r}_i;\mathbf{a}) = 0$, the observed displacements only being caused by measurement noise. The value of χ^2 is 58.7 and based on a chi-square test we reject this hypothesis, its probability being less than 0.001.

Ground deformation caused by the removal of magma from a subsurface reservoir often resembles the surface deformation caused by a pressure decrease in a spherical volume within an elastic halfspace. Considering the sphere as a point source (a/d<<1 where a is the radius of the volume and d is the depth from the surface to the center of the volume) the deformation is given by the Mogi equations [*Mogi*, 1958] (first formulated by *Anderson* [1936]). The horizontal displacement at location $\mathbf{r}_i$ (latitude and longitude) is

$$\delta \mathbf{r}_i^{pre}(\mathbf{r}_i;\mathbf{a}) = C\frac{\mathbf{r}_i - \mathbf{r}_o}{((\mathbf{r}_i - \mathbf{r}_o)^2 + d^2)^{3/2}} \qquad (2)$$

where $\mathbf{r}_o$ is horizontal location of the source, d is its depth, and C is its strength equal to $3a^3\Delta P/4\mu$ where ΔP is the change of fluid pressure in the sphere and μ is the modulus of rigidity of the crust. The Mogi equations provide a successful description of deformation near volcanoes since they are valid where $(a/d)^5$<<1 [*McTigue*, 1987], rendering many magma chambers effective point sources at typical observation ranges.

The best fitting parameters of a Mogi model for the Hekla magma chamber were sought by a grid search method. Displacements caused by a hypothetical point source at varying depths (1-20 km) and locations (in a 20×20 km^2 area) in the vicinity of Hekla were calculated. This was realized by: i) fixing the depth of the source, ii) establish a location grid for hypothetical sources, iii) search for the best fitting value of C for each source location by comparing hypothetical and observed displacements, iv) calculate the corresponding value of χ^2. The process was then repeated for a new point source depth. The best fitting source is located at 63.995°N $^{+4}_{-3}$ km and 19.69°W $^{+1.5}_{-2}$ km at a depth of 9 $^{+6}_{-7}$ km with $-2\pi C$ = 0.1 $^{+0.08}_{-0.04}$ km^3. The parameter bounds are 1σ uncertainties, estimated from the distribution of χ^2. We assume a change in one of the model parameters a_i by amount Δa_i, and optimize all the other parameters for minimum χ^2. Then $\Delta\chi^2 = \chi^2(a_i + \Delta a_i) - \chi^2_{min} = 1$ when Δa_i equals one standard deviation for the parameter a_i [*Bevington*, 1969, p. 243]. The best fitting model displacements are shown in Figure 3. The most probable location of the source is about 1 km WNW of the summit of Hekla but uncertainties are such that the source can be anywhere beneath the volcanic cone (Figure 2). The volume of the surface subsidence bowl, equal to $2\pi|C|$, is 0.1 $^{+0.08}_{-0.04}$ km^3. The absence of GPS control close to the volcano limits the information we can extract from our data about the source depth and its strength. At a distance of more than 13 km from the source the predicted displacements caused by a shallow source of relatively low strength are similar to the displacements caused by a deeper source of higher strength (Figure 4), and consequently our uncertainties for the source depth and C are large. A GPS control closer to the volcano is preferred for future deformation monitoring. The minimum value of χ^2 we find is χ^2_{min} = 10.9. A chi-square test indicates that the Mogi model is consistent with the observed deformation, and residual displacements can be caused by measurement errors. Opening of a conduit or dike from the magma chamber to the surface will also cause deformation and the average 2 cm/year total spreading velocity in South Iceland could have caused up to 3 cm left-lateral displacement across the transform zone in the 18 months between the August 1989 and February 1991 surveys. The ability of the Mogi model to explain the deformation and the apparently random residual displacements suggest, however, that pressure decrease within the magma chamber during the eruption was the primary deformation source between the 1989 and 1991 GPS surveys.

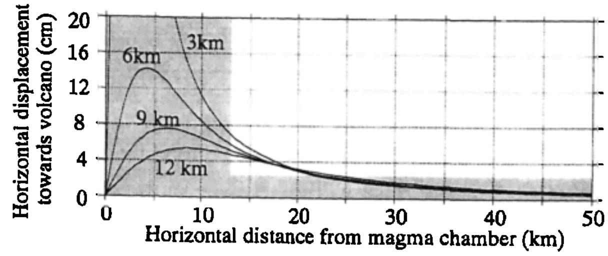

Fig. 4. Displacements caused by a Mogi point source. The four curves are for 3, 6, 9 and 12 km deep sources, with $-2\pi C$ equal to 0.081, 0.084, 0.101 and 0.127 km^3 respectively. The strength parameter C for the fixed depth is the one that best fits the observed deformation. The unshaded region indicates the range of distances (>13 km) considered in this study, and where the deformation exceeds the data uncertainty (2.5 cm).

Discussion

From the Mogi equations in the form expressed by *Anderson* [1936], it is evident that when a<<d, the volume of the surface subsidence bowl is 3/2 times the volume change of the underlying chamber (for Poisson's ratio 0.25). This volume ratio will be reduced somewhat for a large chamber but for $a < 0.5d$ the reduction is less than 10% [*Tryggvason*, 1981]. The $0.1^{+0.08}_{-0.04}$ km^3 estimated surface deflation volume thus corresponds to $\sim 0.067^{+0.05}_{-0.03}$ km^3 magma chamber volume change. The total volume produced in the eruption is estimated 0.15 km^3 [*Gudmundsson et al.*, 1992] more than twice this value. The difference between the ejected lava volume and the magma chamber volume change is $0.083^{+0.03}_{-0.05}$ km^3. This apparent discrepancy could be caused by expansion of the magma in the chamber, the finite size of the chamber, strain changes in the crust around the chamber larger than predicted by the Mogi model, and gas release from the magma due to decompression. We evaluate the first of these possible effects. The volume change, ΔV, of magma in a chamber of volume V_{ch}, associated with a pressure change ΔP, is $\Delta V = -V_{ch}\Delta P/k_{magma}$ where k is the bulk modulus. From the Mogi equations we find $V_{ch}\Delta P = (8/9)\mu_{crust}2\pi|C|$. If magma expansion is the only contribution to the volume discrepancy then $V_{lava} \sim (2/3 + (8/9)\mu_{crust}/k_{magma})2\pi|C|$. The $1.5^{+1}_{-0.7}$ ratio of volume of the lava and the surface subsidence bowl we find implies $\mu_{crust}/k_{magma} = 0.94^{+1.13}_{-0.79}$. The bulk modulus is equal to 17.5 GPa for basaltic andesite melt of similar composition as the 1991 Hekla lava [*Sato and Manghnani*, 1985], leading to 16.5^{+20}_{-14}GPa as an estimate for the rigidity of the crust. This is comparable to the rigidity of the upper crust near Hekla, down to about 5 km depth, estimated from seismic velocities [*Pálmason*, 1971].

A rough estimate of the pressure decrease in the chamber is provided by the observation that in the initial stage of the Hekla eruption, lava was ejected from 600 to 1400 m elevations whereas in the final stage lava was issued from a fissure at 1000 m elevation. The decrease from 1400 to 1000 m elevation from which magma flowed during the eruption is equivalent to 10 MPa assuming a density of 2550 kg/m^3 throughout the magma chamber and vent. A chamber volume of 145^{+52}_{-87} km^3 would lead to $0.083^{+0.03}_{-0.05}$ km^3 chamber volume expansion, explaining fully the volume discrepancy. Since other factors may contribute to the volume discrepancy this is a maximum volume estimate for the Hekla magma chamber, equivalent to a sphere of radius 3.6 km or less.

The 1991 eruption of Hekla was not only monitored with GPS geodesy. Initial interpretation of data from borehole strainmeters near Hekla indicates a pressure decrease in a ~11 km deep magma chamber and compressional strain caused by formation of shallow dikes [*Linde et al.*, 1991]. Initial interpretation of ground tilt measurements in the vicinity of Hekla before and after the eruption suggest a Mogi point source located 4 to 6 km north or northwest of the summit, at a depth of 3.5 - 5 km with $2\pi C$ = -0.1±0.05 km^3 [*Eysteinn Tryggvason*, Nordic Volcanological Institute, personal communication]. The agreement between the GPS far-field horizontal deformation data and the near-field tilt data for the strength parameter is convincing. Distance measurements in 1981-82 after the 1980-81 Hekla eruption were interpreted in terms of inflating magma reservoir about 2 km SSW of the summit, at a depth of 8 km [*Kjartansson and Gronvold*, 1983]. The lava erupted in the 1980-81 and 1991 eruptions are almost identical in composition [*Gudmundsson et al.*, 1992] suggesting that lava expelled in these eruptions came from the same reservoir. The uncertainties in our estimates for the depth and location of the Hekla magma reservoir are such that all the previously estimated locations and depths for the reservoir fall within our 1σ interval for these parameters.

Conclusions

We have presented a successful recording of a magma chamber deflation with a relatively sparse network of GPS control points. The volume of the surface subsidence bowl estimated from the GPS data is $0.1^{+0.08}_{-0.04}$ km^3, consistent with an independent estimate from leveling data. The lava ejected during the 1991 eruption were expelled from a magma chamber beneath the volcano at 9^{+6}_{-7} km depth. The inferred total volume of the magma chamber is roughly 145^{+52}_{-87} km^3, equivalent to a sphere of 3.6 km radius or less. The absence of control points closer to the magma chamber is the main obstacle in deriving a more precise depth for the Hekla magma reservoir. The model presented here is the simplest one possible that is compatible with the GPS data.

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